UpperMark™
Study Handbook

CAIA® Level II

Volume 2

Topic 4: Methods and Models

Topic 5: Accessing Alternative Investments

Topic 6: Due Diligence and Selecting Managers

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It is recommended that candidates use any test preparation product together with the original reading materials suggested in the CAIA Study Guide.[1]

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Preface

Volume 2 of the UpperMark™ *Study Handbooks* provides a comprehensive and concise account of each learning objective (L.O.) in Topics 4-6 of the CAIA Level II Study Guide. The *Study Handbook* is compiled using the reference materials recommended by the CAIA Association and, as in Volume 1, is organized as follows.

- Each Chapter in the Study Guide is presented as a separate chapter, keywords are indicated in ***bold italics***, and learning objective sub-bullets are indicated by underlined, capitalized subheadings (e.g., ROLE OF INVESTMENT OBJECTIVES AND CONSTRAINTS).
- The list of keywords and learning objectives from the Study Guide is provided at the end of each chapter.
- Space is provided at the end of each chapter for you to record your *Personal Study Notes*.
- A set of practice exam questions (multiple-choice and constructed-response) is provided at the end of each chapter. A considerably larger set of practice questions (multiple-choice and constructed-response questions, and constructed-response question sets [“Essays”]) is in our *TestBank* software.

Supplementary information is included in footnotes.

The CAIA Equation Exception List is provided in Appendix 2 of this Study Handbook. As in Volume 1, the first occurrence of each equation on the list is followed by a (*G*) to indicate that it will be *given* on the exam.

We wish you the best with your exam preparation.

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Topic 4

Methods and Models

Topic 4 is composed of seven Readings on different types of financial models, which are essential tools for portfolio management.

1. Reading 4.1 broadly overviews a number of models, including interest rate models and credit risk models.
2. Reading 4.2 describes the binomial tree method for valuation and hedging.
3. Reading 4.3 reviews multi-factor equity pricing models.
4. Reading 4.4 discusses directional investment strategies in which investors alter risk exposures to time market moves based on technical analysis.
5. Reading 4.5 describes multivariate empirical methods (e.g., multiple regression).
6. Reading 4.6 discusses relative value methods with a focus on fundamental analysis.
7. Reading 4.7 discusses, in the context of real estate, the tax consequences of depreciation and transaction- and appraisal-based indices.

Reading 4.1

Modeling Overview and Fixed-Income Models

Financial models express relations that underlie financial instruments and markets, and describe concepts underlying investment and risk management strategies. This Reading describes different types of models, with a focus on interest rate and credit risk models. The interest rate models described are equilibrium, arbitrage-free, and binomial models. The credit risk models described are the structural and reduced-form models, which use different economic approaches to generate the probability of default and credit spread; and a third model that uses a borrower's financial data to generate a credit score for the borrower.

Learning Objectives

4.1.1 Demonstrate knowledge of underlying models of investment strategy.

i. Compare normative models with positive models
ii. Distinguish between theoretical and empirical models
iii. Distinguish between applied versus abstract models
iv. Compare cross-sectional versus time-series models
v. Discuss the importance of methodology in model building

Keywords

1. Abstract models
2. Applied models
3. Cross-sectional models
4. Empirical models
5. Endogenous variable
6. Exogenous variable
7. Normative model
8. Panel data sets
9. Positive model
10. Theoretical models
11. Time-series models

4.1.2 Demonstrate knowledge of equilibrium models and arbitrage-free models of the term structure.

i. Contrast equilibrium fixed-income models with arbitrage-free models
ii. Apply Vasicek's model
iii. Contrast the Cox, Ingersoll, and Ross (CIR) model with Vasicek's model
iv. Discuss the Ho and Lee model

Keywords

1. Arbitrage-free models of the term structure
2. Cox, Ingersoll, and Ross model
3. Equilibrium models of the term structure
4. Ho and Lee model
5. Vasicek's model

4.1.3 Demonstrate knowledge of the Black-Derman-Toy (BDT) model.

i. Interpret a binomial BDT tree
ii. Understand how to calibrate the level of rates based on average returns
iii. Understand how to calibrate the spread of rates based on volatilities
iv. Discuss BDT calibrations in general

Keyword

1. Black-Derman-Toy Model (BDT model)

4.1.4 Demonstrate knowledge of credit risk and credit risk modeling.

i. Distinguish types of credit events that may lead to an increase in credit risk
ii. Explain exposure at default (EAD) and loss given default (LGD)
iii. Describe how adverse selection and moral hazard relate to credit risk
iv. Discuss how probability of default (PD) and recovery rate (RR) affect credit risk
v. Calculate loss given default and expected loss from credit risk
vi. Describe the basic concepts of credit risk modeling
vii. Contrast the three approaches to credit risk modeling

Keyword

1. Credit events

4.1.5 Demonstrate knowledge of the structural model approach to credit risk.

i. Describe Merton's structural model using the option-like nature (both call options and put options) of traditional corporate securities
ii. Describe the inherent conflict of interest that exists between shareholders and bondholders
iii. Evaluate advantages and disadvantages of the Merton Model
iv. Understand how binomial trees can be used to value structured products

4.1.6 Demonstrate knowledge of the Merton model.

i. Apply the Merton model to determine equity values and payoffs to bondholders for a given investment
ii. Calculate the value of risky debt using the Black-Scholes option pricing model in the Merton model to price a given firm's equity as a call option on the stock of the underlying company
iii. Evaluate the use of Black-Scholes option pricing in the Merton model
iv. Analyze the role of credit spreads in structural models and how the credit spread can be used to calculate the price of risky debt
v. Understand the four important properties of the Merton model

4.1.7 Demonstrate knowledge of the Kealhover, McQuown, and Vasicek (KMV) credit risk model.

i. Describe the characteristics and application of the KMV model
ii. Evaluate the credit score (the distance to default) for a given firm using the KMV model
iii. Evaluate the expected default frequency for a given investment using the KMV model

Keywords

1. Default trigger
2. Distance to default (DD)
3. Expected default frequency (EDF)
4. KMV model

4.1.8 Demonstrate knowledge of reduced-form models.

i. Describe the characteristics of reduced-form models
ii. Discuss the role of default intensity in reduced-form models and calculate default intensity for a given firm
iii. Demonstrate how default intensity can be incorporated into the valuation of risky debt
iv. Analyze the relationship among credit spreads, default intensities, and recovery rates, and use two of these factors as variables to solve for the third for a given investment
v. Describe the two predominant reduced-form credit models

Keyword

1. Default intensity

4.1.9 Demonstrate knowledge of empirical credit models.

i. Contrast empirical credit models with structural and reduced-form models
ii. Describe the purpose and characteristics of the Altman Z-score model
iii. Understand the five financial ratios that are used as inputs to determine Altman Z-scores
iv. Evaluate Z-scores in Altman's credit scoring model

Keywords

1. Credit score
2. Z-score model

L.O. 4.1.1 **DEMONSTRATE KNOWLEDGE OF UNDERLYING MODELS OF INVESTMENT STRATEGIES.**

Variables used in models may be classified as exogenous or endogenous.

- An ***exogenous variable*** is a value that is determined outside a model and thus taken as given.
- An ***endogenous variable*** is determined inside a model and thus takes on the value the model prescribes.

For instance, in an endowment fund's cash management model, an exogenous variable may be the amount of cash received from donations and income from investments, and endogenous variables may be decision variables such as the amount of money invested in new deals.

By understanding different types of models, asset allocators may form portfolios that are better diversified across types of models. Four common distinctions of models are discussed in this L.O.: 1) normative vs. positive, 2) theoretical vs. empirical, 3) applied vs. abstract, and 4) cross-sectional vs. time-series.

1. <u>NORMATIVE VS. POSITIVE MODELS</u>

   Models may be classified as normative or positive.

   - ***Normative models***
     - Normative economic models aim to describe how market participants and asset prices should behave. For instance, arbitrage-free models describe relationships that should hold given that arbitrageurs' actions will eliminate arbitrage opportunities. For example, a normative strategy is a strategy based on put-call parity.
     - These models are used to identify driving factors of rational financial decisions based on idealized assumptions and conditions.
     - They may also be used to identify potential mispricings by identifying how assets should be priced. Normative reasoning assumes that actual prices converge toward prices predicted by a normative model.
   - ***Positive models***
     - Positive economic models explain/predict how market participants and asset prices actually behave. For instance, technical trading is based on positive models. For example, a positive strategy is a strategy based on point-and-figure charts.[1]
     - These models are often used to identify potential mispricings by identifying patterns in actual price movements.

   The primary criterion for assessing models is their ability to predict the future. The reality of assumptions or ability to explain the past is of secondary importance. Normative and positive models can be used to predict future behavior and for analyzing alternative investments. Arguably, the key aspect of analyzing many trading strategies is knowing whether the underlying model is based on how prices should or do behave. For instance, if a fund manager implements a trade to benefit from a forecasted change in prices, did the manager base the forecast on how prices should behave or by observing how past prices behaved?

[1] Point-and-figure charts describe price movements without regard for the passage of time. The charts consist of columns of X's and O's, where the X columns represent rising prices and the O columns represent falling prices. No change in a point-and-figure chart indicates no movement in price.

2. THEORETICAL VS. EMPIRICAL MODELS

Related to normative and positive models are theoretical and empirical models. The choice between a theoretical and an empirical model generally depends on the complexity of the underlying relationships and the reliability of the data.

- Theoretical models
  - These models describe behavior based on assumptions that reflect well-established underlying behavior.
  - They provide a reasonable explanation of simple behavior, but are not practical for securities with complex attributes and relationships. A single theoretical model does not exist that can explain all relationships in different markets.
  - An application of theoretical models includes theoretically determining the price of an option based on assumptions such as perfect markets, stock prices that follow a particular process, and absence of arbitrage.
- Empirical models
  - These models describe behavior based on observations of historical data. They require underlying variables to be relatively constant or to change in a predictable way. They also require large data sets to produce reasonable results.
  - Empirical models are often used to explain complex behavior. As such, they are most effective for alternative investments due to the investments' illiquidity, time-varying risks, and use of dynamic strategies.
  - Applications of empirical models include analyzing complex securities with option features and approximating the relationship between observed prices of options and their underlying variables.

3. APPLIED VS. ABSTRACT MODELS

Models may be distinguished as applied or abstract.

- ***Applied models***
  - These models are used for solving real-world problems. For instance, the Markowitz model of portfolio management is an applied model that provides a useful approach to achieve diversification efficiently.
  - Most asset pricing models used in traditional and alternative investing are applied.
- ***Abstract models*** (or basic models)
  - These models are typically theoretical and explain hypothetical behavior in unrealistic situations. They do not address real-world problems.
  - For instance, an abstract model might describe how two people trade securities in a world with only two people and two risk factors.

4. CROSS-SECTIONAL VS. TIME-SERIES MODELS

Models may be classified as cross-sectional or time-series models.

- ***Cross-sectional models***
  - These models analyze relationships across variables observed at a single point in time (e.g., using investment returns to explain differences in risk premiums).
- ***Time-series models***
  - These models analyze the behavior of an asset or a set of assets across time.

Models may also be both cross-sectional and time-series. This type of model is referred to as a panel study and using data composed of multiple assets over multiple time periods. The data are referred to as ***panel data sets***, cross-sectional time-series data sets, or longitudinal data.

For instance, time-series and cross-sectional models may be used to analyze returns on real estate investment trusts (REITs) and on a REIT index.

- A time-series model may be constructed that explains the REIT index returns over time by regressing the index returns against mortgage rates and stock returns. A cross-sectional model is then used to explain why various REITs have different returns by regressing individual REIT returns against variables such as region and property type.
- Alternatively, all returns for each time period and for each REIT can be analyzed using a single regression model in a panel study.

Some asset pricing models may be classified in more than one way. For instance, abstract models tend to be normative and theoretical, while applied models tend to be empirical and positive. In some studies, complementary modeling approaches may be combined. For instance, a theoretical model can be designed and then tested in an empirical framework. Other examples of models with multiple distinctions are provided below.

- Theoretical, normative, and applied model – An analyst identifies a profitable trading opportunity by specifying an asset's equilibrium price and recommending trades when the asset's actual price deviates from its equilibrium price.
- Empirical, positive, and applied model – An analyst identifies a statistical trading pattern and uses the pattern to generate trading signals.

L.O. 4.1.2 DEMONSTRATE KNOWLEDGE OF EQUILIBRIUM MODELS AND ARBITRAGE-FREE MODELS OF THE TERM STRUCTURE

Models of the term structure of interest rates are used to describe the evolution of default-free bond values and to value fixed-income derivatives. There are two broad approaches to modeling the term structure of interest rates: equilibrium models and arbitrage-free models. This L.O. focuses on these models.

Equilibrium models of the term structure (also referred to as first-generation models) make assumptions about the structure of fixed-income markets and then model bond prices and the term structure of interest rates based on economic reasoning. The equilibrium models discussed in this L.O. model the entire term structure of interest rates (i.e., the yield on long-term bonds) by taking the short-term interest rate process as given (i.e., assuming a process for the short-term rate) and assuming that the unbiased expectation hypothesis holds for bond prices (which implies that credit risk-free bonds of all maturities have the same expected return over the short term).

The first section of this L.O. discusses two equilibrium models: Vasicek (1977) and Cox, Ingersoll, and Ross (1985).

VASICEK MODEL

Vasicek's model is a single-factor model of the term structure that assumes constant volatility and that the short-term interest rate drifts toward a specific long-term mean. The model describes the mean-reverting process for the short-term interest rate as:

$$\tilde{r}_{t+1} = r_t + \kappa(\mu - r_t) + \sigma\tilde{\varepsilon}_{t+1},$$

where $\tilde{r}_{t+1}$ is the next period's short-term rate, r_t is the current short-term rate, μ is a positive constant that represents the short-term rate's long-term mean value, κ is a positive constant that determines the speed of adjustment to the long-term mean, σ is the volatility of changes in interest rates, and $\kappa(\mu - r_t)$ and $\sigma\tilde{\varepsilon}_{t+1}$ are two adjustments.

- The first adjustment $\kappa(\mu - r_t)$ indicates that the next period's short-term rate will be higher than the current rate if $\mu > r_t$.
  - Since the short-term rate is mean-reverting (i.e., reverting to its long-term mean), the short-term rate is likely to increase if it is currently below its long-term mean and likely to decrease if it is above its long-term mean.
  - The larger the κ, the faster the short-term rate approaches its long-term mean.
- The second adjustment factor $\sigma\tilde{\varepsilon}_{t+1}$ represents the unexpected change in the short-term rate, where $\tilde{\varepsilon}_{t+1}$ represents a noise factor that is assumed to be a standardized normally distributed random variable (i.e., with a mean of zero and a standard deviation of one).

  The Vasicek model may be expressed in terms of the next period's expected short-term rate as:

$$E[r_{t+1}] = r_t + \kappa(\mu - r_t).$$

This shows that the expected change in the short-term rate is given by $\kappa(\mu - r_t)$.

A criticism of the Vasicek model is that it assumes the volatility of changes in interest rates is constant as the level of interest rates changes (e.g., volatility of interest rate changes is always 1.2%). A consequence of this that the model may produce negative interest rates, which is another criticism of the model.

Example

In the Vasicek model, the long-term mean of the short-term interest rate is 6% and the speed of adjustment parameter is 0.75. If the current short-term rate is 4.3%, what is the expected short-term rate in the next period?

Expected short-term rate in next period is:

$$E[r_{t+1}] = r_t + \kappa(\mu - r_t) = 4.3\% + 0.75(6\% - 4.3\%) = 5.575\%.$$

Vasicek model and the term structure of interest rates

In the Vasicek model, all bond prices are driven by the short-term interest rate, which implies that the only source of uncertainty in the bond market is the random change in the short-term rate. Formulas for bond prices may be determined in different ways. For instance, assuming that the unbiased expectations hypothesis holds for the term structure, all credit-risk-free bonds are expected to earn the same rate of return in the short-term. Based on this assumption, a formula can be derived for the yield to maturity of a zero-coupon bond and for the term structure of interest rates (or yield curve) in the Vasicek model.

The Vasicek model can generate term structures of different shapes: downward-sloping, upward-sloping, and humped. The slope of the curve is driven by the relationship between the short-term rate and the long-term mean rate. The hump reflects investors' risk aversion.

COX, INGERSOLL, AND ROSS MODEL

The ***Cox, Ingersoll, and Ross*** (CIR) ***model*** modifies the Vasicek model so that the variance of the short-term rate is proportional to the short-term rate.[2] As a result, the CIR model does not allow negative interest rates, since, as rates approach zero, their volatility approaches zero. The CIR model is a single-factor model that describes the short-term interest rate process as:

$$r_{t+1} = r_t + \kappa(\mu - r_t) + \sqrt{r_t}\sigma\tilde{\varepsilon}_{t+1}, \qquad \textbf{(G)}^3$$

where the three constant parameters κ, μ, and σ are as defined in the Vasicek model.

- The relationship that the variance of the short-term rate is proportional to the rate itself is observed empirically: volatility of interest rate changes is higher when the short-term rate is relatively high.
- Like the Vasicek model, the CIR model can generate yield curves of different shapes.

ARBITRAGE-FREE MODELS OF THE TERM STRUCTURE

Arbitrage-free models of the term structure (also referred to as second-generation models) generate bond prices that do not allow for arbitrage opportunities. Under risk-neutral modelling, returns on all investments should equal the short-term rate. Arbitrage-free models also use the currently observed term structure to determine the parameters of the model, which results in a theoretical term structure model that is consistent with the observed term structure. As a result, any fixed-income derivative instrument priced using this model will be consistent with the current term structure and will preclude arbitrage opportunities involving the derivatives and available bonds.

This section describes the Ho and Lee (1986) arbitrage-free model, which was the first arbitrage-free model of interest rates developed.

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[2] In other words, volatility equals σ in the Vasicek model and equals $\sqrt{r_t}\sigma$ in the CIR model.

[3] As stated in the Preface, the CAIA Association may provide specific equations on the exam. Equations that may be provided are indicated in our Handbooks with a (*G*) and are in Appendix 2 of this *Handbook*.

HO AND LEE MODEL

The ***Ho and Lee model*** is a single-factor model that assumes that the short-term interest rate follows a normally distributed process, with a drift parameter selected so that the modeled term structure of interest rates fits the observed term structure. The Ho and Lee model describes the short-term rate as:

$$r_{t+1} = r_t + \theta_t + \sigma\tilde{\varepsilon}_{t+1}\,, \qquad (\boldsymbol{G})$$

where θ_t is a time-dependent mean change in the short-term rate (selected to ensure that the model fits the initial term structure of interest rates), σ is the constant standard deviation of changes in the short-term rate, and $\tilde{\varepsilon}_{t+1}$ is a binomial random variable (which is either +1 or –1). In contrast to Vasicek's model that has constant parameters, the θ_t parameter in the Ho and Lee model is not constant, but is determined by the current term structure of interest rates.

Ho and Lee use this model to determine bond prices using a binomial pricing approach, in which current zero-coupon bond prices are taken as given and used to value the parameters of the model based on the current term structure of interest rates. The term structure is then assumed to be affected by random changes in interest rates. Bond prices evolve in response to random changes in interest rates. The model uses the concept that, with risk-neutral probabilities, the bond price in every state equals the bond's expected value in the next period discounted at the riskless rate to obtain analytical solutions for the bond price in each future state.

Since the Ho and Lee model is calibrated to fit the currently observed term structure of interest rates, resulting prices of interest rate instruments (e.g., callable bonds and bond options) are consistent with the current term structure. This implies that the actions of arbitrageurs can ensure that derivative prices are tied to observable bond prices so that arbitrage profits are not possible.

The key disadvantages of the Ho and Lee model are that it assumes a simple binomial process for bond prices and it can produce negative interest rates. Variations of this model have been developed that prevent negative interest rates and use more sophisticated processes for bond prices.

L.O. 4.1.3 **DEMONSTRATE KNOWLEDGE OF THE BLACK-DERMAN-TOY (BDT) MODEL.**

The ***Black-Derman-Toy (BDT) model*** (Black and Toy 1990) is an interest rate model that constructs no-arbitrage interest rate trees using both the observed term structure of interest rates and rate volatilities (i.e., implied volatilities of interest rate caplets). The model may be used with any compounding assumption to model spot rates, forward rates, and/or discount factors, and used to find no-arbitrage values of fixed-income derivatives. As described below, the BDT model focuses on two relations: average forward rates and interest rate volatilities.

BDT BINOMIAL TREE

A BDT binomial tree represents short-term (1-year) spot rates, where, at each rate in the tree, there are two possible rates next year that each occur with a probability of 0.5 (or 50%). For instance, as depicted, in Figure 1, for a 2-year binomial tree with a current 1-year spot rate of r_0, there are two possible 1-year spot rates next year: r_u in the up node and r_d in the down node.